LOYOLA COLLEGE (AUTONOMOUS) CHENNAI – 600 034

U.G. DEGREE EXAMINATION – ALLIED

FIRST SEMESTER – NOVEMBER 2024

UMT 1303 –
MATHEMATICS FOR STATISTICS, MATHEMATICS FOR STATISTICS - I

Date: 20-11-2024   Dept. No.   Max. : 100 Marks
Time: 09:00 am-12:00 pm

SECTION A - K1 (CO1)	
Answer ALL the Questions -	**(10 x 1 = 10)**
1.	**Answer the following.**
a)	Recall chain rule in differentiation.
b)	State Leibnitz theorem.
c)	Write the partial differential co-efficient of $u = \sin(ax + by + cz)$ with respect to x.
d)	Determine the integral value of $x\sin x$ with respect to x.
e)	Point out the integral value of e^{-5x} with respect to x.
2.	**Fill in the blanks.**
a)	When differentiating the equation $x^2 + y^2 = 25$ implicitly with respect to , we get $\dfrac{dy}{dx}$ =………
b)	A critical point is where the derivative of a function is either zero or …………….. .
c)	The partial derivative of $f(x, y) = x^2 y + 3x y^2$ with respect to x is …………..
d)	The integral of $\dfrac{1}{\sqrt{2x}}$ with respect to x is ………….
e)	The linear property of definite integral is ………..
SECTION A - K2 (CO1)	
Answer ALL the Questions	**(10 x 1 = 10)**
3.	**Choose the correct answer for the following.**
a)	What is the derivative of $f(x) = \sin 10x$? (i) $\sin 10x$ (ii) $-\sin 10x$ (iii) $-10\cos 10x$ (iv) $10\cos 10x$
b)	Leibnitz theorem provides a formula to find (i) The nth derivative of a sum of two functions (ii) The nth derivative of a product of two functions (iii) The integral of a product of two functions (iv) The integral of a sum of two functions
c)	The second derivative test is used to determine (i) The existence of limits (ii) The concavity of a function (iii) The continuity of a function (iv) The average value of a function

d)	What technique is commonly used to integrate rational algebraic functions of the form $\frac{P(X)}{Q(X)}$, where $P(X)$ and $Q(X)$ are polynomials? (i) Trigonometric substitution (ii) Integration by parts (iii) Partial Fractions (iv) Substitution Method
e)	The definite integral of $x^3(1-x)$ from 0 to 1 is ………….. (i) $\frac{-1}{20}$ (ii) 1 (iii) $\frac{1}{20}$ (iv) 20
4.	**State true or false.**
a)	If a function $f(x)$ has a derivative at a point, it is guaranteed to be differentiable everywhere.
b)	A function is increasing on an interval if its first derivative is positive on that interval.
c)	The derivative of a constant function is zero.
d)	Integration by parts is a technique based on the product rule of differentiation.
e)	If $F(X)$ is an antiderivative of $f(x)$, then $\int\limits_a^b f(x)dx = F(b)+F(a)$.

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SECTION B - K3 (CO2)

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Answer any TWO of the following   **(2 x 10 = 20)**

5.	Apply Leibnitz theorem to compute n^{th} derivative of $x^2 e^{-10x}$.
6.	Evaluate y_n when $y = \dfrac{3}{(x-1)(2x-5)(x+1)}$ by resolving into partial fractions.
7.	Determine $\int \dfrac{x^{24}}{x^{10}+1}\,dx$.
8.	Using partial fraction method, evaluate $\int \dfrac{dx}{(1+x)(1+x^2)}$.

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SECTION C – K4 (CO3)

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Answer any TWO of the following   **(2 x 10 = 20)**

9.	Verify Euler's theorem when $u = x^3 - 3x^2 y + 3xy^2 + y^3$.
10.	Determine $\dfrac{du}{dt}$ where $u = x^2 + y^2 + z^2, x = e^t, y = e^t \sin t$ and $z = e^t \cos t$.
11.	Evaluate $\int\limits_0^{\frac{\pi}{2}} \log \text{¿¿}$
12.	Compute $\int \dfrac{x^2 + 2x - 1}{\sqrt{x}}\,dx$.

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SECTION D – K5 (CO4)

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Answer any ONE of the following   **(1 x 20 = 20)**

13.	Determine the maxima and minima of the function $x^3 - 18x^2 + 96x + 4$.
14.	State the condition under which a rational function is integrable and evaluate $\int \dfrac{3x+1}{(x-1)^2(x+3)}\,dx$.

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SECTION E – K6 (CO5)

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Answer any ONE of the following   **(1 x 20 = 20)**

15.	If $y = \sin(m \sin^{-1} x)$, prove that $(1-x^2) y_2 - x y_1 + m^2 y = 0$ and hence show that $(1-x^2) y_{n+2} - (2n+1) x y_{n+1} + (m^2 - n^2) y_n = 0 \cdot$
16.	(i) Compile reduction formula for $\int \sin^n x \, dx$ and hence evaluate $\int_0^{\frac{\pi}{2}} \cos^8 x \, dx$. (10 marks) (ii) Test that $\int_0^{\frac{\pi}{2}} ¿¿¿¿ \cdot$ (10 marks)

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